

# Risk Assessment for Banking Systems\*

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# Risk Assessment for Banking Systems

## **Abstract**

In this paper we suggest a new approach to risk assessment for banks. Rather than looking at them individually we analyze risk at the level of the banking system. Such a perspective is necessary because the complicated network of mutual credit obligations can make the actual risk exposure of the entire system invisible at the level of individual institutions. We apply our framework to a cross section of individual bank data as they are usually collected at the central bank. Using standard risk management techniques in combination with a network model of inter-bank exposures we analyze the consequences of macro-economic shocks for bank insolvency risk. In particular we consider interest rate shocks, exchange rate and stock market movements as well as shocks related to the business cycle. The feedback between individual banks and potential domino effects from bank defaults are taken explicitly into account. The model determines endogenously probabilities of bank insolvencies, recovery rates and a decomposition of insolvency cases into defaults that directly result from movements in risk factors and defaults that arise indirectly as a consequence of contagion.

**Keywords:** Systemic Risk, Inter-bank Market, Financial Stability, Risk Management

**JEL-Classification Numbers:** G21, C15, C81, E44

# 1 Introduction

Measuring credit risk for banks is particularly challenging because of the importance of financial linkages in the banking system. Direct knock on effects of corporate defaults on other corporations through financial linkages will typically be fairly negligible. The situation is different for banking systems. The financial network of mutual credit obligations stemming from liquidity management, re-financing, hedging, and security trading creates a potential for contagious insolvencies or domino-effects on top of the common exposure problems. To get a reliable assessment of credit risk for banking systems this network structure has to be taken into account.

For regulators there are two major reasons why the correct measurement of credit risk in the inter-bank market is of particular interest. First, like all other assets, inter-bank loans have to be backed with equity capital. To determine the correct capital requirement, default as well as recovery rates have to be estimated. Under current regulations, inter-bank loans have lower capital requirements than commercial loans, implicitly assuming that credit risk is lower in the inter-bank market. In our paper we suggest a new methodology to estimate default and recovery rates. Second, regulators are concerned about systemic risk in the banking sector and the possibility of a chain reaction of bank defaults. Safeguarding the banking system against a systemic crises is one of the major rationales for banking supervision and regulation<sup>1</sup>. We argue that monitoring systemic risk requires an analysis at the level of the banking system rather than at the level of individual banks. To implement this system perspective bank supervisors have to take into account the risks stemming from financial linkages between banks.

In our paper we propose a new method to model the inter-bank-network explicitly. We have access to a unique dataset provided by the Austrian Central Bank (OeNB) with detailed information on inter-bank liabilities for a whole banking system. We also have access to market risk exposures as well as detailed information on the banks' loan portfolio composition. Thus, we can estimate default frequencies and recovery rates for the banking system and investigate the stability of the banking system with respect to systemic risk. To our best knowledge this is the first attempt to utilize such a comprehensive dataset for the risk analysis of an entire banking system.

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<sup>1</sup>Greenspan (1997) notes on the FED's agenda: "Second only to its macro-stability responsibilities is the central bank's responsibility to use its authority and expertise to forestall financial crises (including systemic disturbances in the banking system) and to manage such crises once they occur."

The general idea of the model is to combine traditional risk management analysis with a network analysis of the inter-bank market. Economic risk scenarios (interest rate shocks, FX movements, loan losses, stock price changes) are modeled by standard risk management tools. All banks are exposed to the same shock simultaneously and the full implications of such an economic shock on the banking system are then analyzed via the network model. If a bank's equity is impaired by a shock and the bank is not able to fully repay its inter-bank loans the propagation of such a shock through the network of mutual credit obligations can be studied. By this approach we are able to quantify the potential for contagious defaults among banks and disentangle it from risk that directly comes from market and non-inter-bank credit exposures. The network model also allows us to compute endogenous default and recovery rates that are consistent with clearing on the inter-bank market. Our approach does not rely on a history of observed bank defaults. Apart from the problem that historical bank default rates are distorted because many troubled banks might be saved by regulatory intervention, defaults due to a systemic crisis are rarely observed. Our analysis explicitly addresses and quantifies the threat of a systemic crisis which may be underestimated when relying only on a history of observed bank defaults.

Comparing the probabilities of fundamental and contagious defaults, we find that for our data set the banking system is fairly stable with respect to contagion. We find that the mean default probability is 0.8% and the probability of contagious default for the average bank is only 0.0062%. Thus, in our sample contagious defaults are relatively unlikely. Even though contagious defaults occur rarely, there are scenarios with many contagious defaults. In our simulation we find scenarios where contagion accounts for up to 75% of all banks defaults. Contagion is a low probability-high impact event.

Previous simulation studies looking at inter-bank exposures such as Humphery (1986), Angelini, Maresca, and Russo (1996), Furfine (2003), and Upper and Worms (2002) investigate contagious defaults that result from the hypothetical failure of some single institution. Such an analysis is able to capture the effect of idiosyncratic bank failures (e.g. because of fraud). We take these studies a decisive step further by combining the analysis of inter-bank connections with a simultaneous study of the banking system's overall risk exposure. Thus, we analyze how adverse economic developments will affect individual institutions and how these shocks are propagated by financial linkages. Instead of basing banking risk analysis on ad hoc individual institution failure scenarios we study risk scenarios for the banking system which are created using standard risk management

techniques. Our model can therefore be seen as an attempt to judge the risk exposure of the system as a whole. A '*system perspective*' on banking supervision has for instance been actively advocated by Hellwig (1997). Andrew Crockett (2000) has even coined a new word - *macro-prudential* - to express the general philosophy of such an approach.<sup>2</sup>

The results of our analysis have policy implications for macro-prudential bank regulation. First, we can see that the probability of contagious defaults depends on bankruptcy costs in a non linear way. An efficient bankruptcy procedure is therefore of crucial importance in the prevention of systemic risk. Second, our model allows us to estimate the reserves for the lender of last resort that are necessary to prevent contagious defaults. In this sense we can compute a "value-at-risk" capital requirement for the regulator. We find that surprisingly little funds have to be set aside to prevent contagious bank failures. About 0.003% of the total assets in the banking system are sufficient to prevent contagion in 99% of the scenarios. However, substantial financial effort is required to prevent fundamental defaults. Here, the regulator needs to set aside 83 times more at the same confidence level.

The rest of the paper is organized as follows. Section 2 describes the network model of the inter-bank market and Section 3 illustrates the sample. The two components of the simulation analysis, the structure of the inter-bank liabilities and the generation of economic scenarios are described in Sections 4 and 5, respectively. Section 6 presents the results of the simulation and driving forces of contagion are discussed in Section 7. Finally Section 8 concludes.

## 2 A Network Model of the Inter-bank Market

The conceptual framework we use to describe the system of inter-bank credits has been introduced to the literature by Eisenberg and Noe (2001). These authors study a centralized static clearing mechanism for a financial system with exogenous income positions and a given structure of bilateral nominal liabilities. We build on this model and extend it to include uncertainty.

Consider a finite set  $\mathcal{N} = \{1, \dots, N\}$  of banks. Each bank  $i \in \mathcal{N}$  is characterized by a given value  $e_i$  net of inter-bank positions and nominal liabilities  $l_{ij}$  against other banks

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<sup>2</sup>See also Borio (2002).

$j \in \mathcal{N}$  in the system. The entire banking system is thus described by an  $N \times N$  matrix  $L$  and a vector  $e \in \mathbb{R}^N$ . We denote this system by the pair  $(L, e)$ .

If for a given pair  $(L, e)$  the total net value of a bank becomes negative, the bank is insolvent. In this case it is assumed that creditor banks are rationed proportionally. Following Eisenberg and Noe (2001) we can formalize proportional rationing in case of default as follows: Denote by  $d \in \mathbb{R}_+^N$  the vector of total obligations of banks towards the rest of the system i.e., we have  $d_i = \sum_{j \in \mathcal{N}} l_{ij}$ . Proportional sharing of value in case of insolvency is described by defining a new matrix  $\Pi \in [0, 1]^{N \times N}$  which is derived from  $L$  by normalizing the entries by total obligations.

$$\pi_{ij} = \begin{cases} \frac{l_{ij}}{d_i} & \text{if } d_i > 0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

We describe a financial system as a tuple  $(\Pi, e, d)$  for which we define a so called *clearing payment vector*  $p^*$  that respects limited liability of banks and proportional sharing in case of default. It denotes the total payments made by the banks under the clearing mechanism.

**Definition 1** *A clearing payment vector for the system  $(\Pi, e, d)$  is a vector  $p^*$  such that for all  $i \in \mathcal{N}$*

$$p_i^* = \min \left[ d_i, \max \left( \sum_{j=1}^N \pi_{ji} p_j^* + e_i, 0 \right) \right] \quad (2)$$

Thus, the clearing payment vector directly gives us two important insights. First: For a given structure of liabilities and bank values  $(\Pi, e, d)$  it tells us which banks in the system are insolvent ( $p_i^* < d_i$ ). Second: It tells us the recovery rate for each defaulting bank ( $\frac{p_i^*}{d_i}$ ).

To find a clearing payment vector we employ the fictitious default algorithm developed by Eisenberg and Noe (2001). They prove that under mild regularity conditions a unique clearing payment vector for  $(\Pi, e, d)$  always exists. These results extend - with slight modifications - to our framework as well.<sup>3</sup>

From the solution of the clearing problem, we can gain additional economically important information with respect to systemic stability. Default of bank  $i$  is called fundamental

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<sup>3</sup>In Eisenberg and Noe (2001) the vector  $e$  is in  $\mathbb{R}_+^N$  whereas in our case the vector is in  $\mathbb{R}^N$ .

if bank  $i$  is not able to honor its promises under the assumptions that all other banks honor their promises.<sup>4</sup>, i.e.

$$\sum_{j=1}^N \pi_{ji} d_j + e_i - d_i < 0$$

If bank  $i$  defaults only because other banks are not able to keep their promises we call this a contagious default, i.e.

$$\sum_{j=1}^N \pi_{ji} d_j + e_i - d_i \geq 0 \text{ but } \sum_{j=1}^N \pi_{ji} p_j^* + e_i - d_i < 0$$

To use this model for risk analysis we extend it to an uncertainty framework by assuming that  $e$  is a random variable.<sup>5</sup> As there is no closed form solution for the distribution of  $p^*$  given the distribution of  $e$  we have to resort to a simulation approach. Each draw  $e$  is called a scenario. By the theorem of Eisenberg and Noe (2001) we know that there exists a (unique) clearing payment vector  $p^*(e)$  for each scenario. Thus from an ex-ante perspective we can assess expected default frequencies from inter-bank credits across scenarios as well as the expected severity of losses from these defaults given we have an idea about the distribution of  $e$ . Furthermore we are able to decompose insolvencies across scenarios into fundamental and contagious defaults. A toy example illustrating the procedure is given in Appendix A.

To pin down the distribution of  $e$  we choose the following route: assume that there are two dates:  $t = 0$  which is the *observation date* and  $t = 1$  which is a hypothetical clearing date where all inter-bank claims are settled according to the clearing mechanism. At  $t = 0$  the portfolio holdings of each bank are observed. The inter-bank related exposures constitute the matrix  $L$ . The remaining portfolio holdings consist of loans, bonds, stocks on the asset side and of liabilities to non banks on the liabilities side. These positions are exposed to market and/or credit risk. We assume that the portfolio holdings remain

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<sup>4</sup>Note that our setup implicitly contains a seniority structure of different debt claims of banks. By interpreting  $e_i$  as net value from all bank activities except the inter-bank business we assume that inter-bank debt claims are junior to other claims, like depositors or bond holders. However inter-bank claims have absolute priority in the sense that the owners of the bank get paid only after all debts have been paid. In reality the legal situation is much more complicated and the seniority structure might very well differ from the simple procedure we employ here. For our purpose it gives us a convenient simplification that makes a rigorous analysis of inter-bank defaults tractable.

<sup>5</sup>One could also allow for a stochastic matrix  $L$ . In our analysis we take the nominal face value of inter-bank debt as fixed.

constant. Hence the value of the portfolio at  $t = 1$  depends solely on the realization of the relevant risk factors. To generate a scenario we draw a realization of these risk factors from their joint distribution and revalue the portfolio to get the value of the portfolio  $e$  at  $t = 1$ . Given this scenario the system is cleared and a clearing vector  $p^*(e)$  is determined.

An application of the network model for the assessment of credit risk from inter-bank positions therefore requires mainly two things. First, we have to determine  $L$  from the data. Second, we have to come up with a plausible framework to create meaningful risk scenarios.

### 3 Data

In the following we give a description of our data. Our main sources are bank balance sheet and supervisory data from the monthly filings (MAUS) to the Austrian Central Bank (OeNB) and the database of the OeNB major loans register (Großkreditevidenz, GKE). We use furthermore data on default frequencies in certain industry groups from the Austrian rating agency Kreditschutzverband von 1870. Finally we use market data from Data-stream.

#### 3.1 Bank Data

Banks in Austria file reports on their business activities to the central bank on a monthly basis. On top of balance sheet data MAUS contains a fairly extensive amount of other data that are relevant for supervisory purposes. They include among others numbers on capital adequacy statistics, off balance exposures, times to maturity and foreign exchange exposures with respect to different currencies.

In our analysis we use a cross section from the MAUS database for September 2002 which we take as our observation period. We use these data to determine the matrix  $L$  as well as the portfolio holdings that are not related to the inter-bank business. The data on the inter-bank exposures is not as detailed as we need it. Yet a particular institutional feature of the Austrian banking system helps us with the estimation of bilateral inter-bank exposures. It has a sectoral organization for historic reasons.<sup>6</sup> Three out of seven

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<sup>6</sup>Banks belong to one of seven sectors: joint stock banks, savings banks, state mortgage banks, Raif-

sectors have a multi tier structure with head institutions. Banks have to break down their MAUS reports on claims and liabilities with other banks according to the different banking sectors, head institution, central bank, and foreign banks. This practice of reporting on balance inter-bank positions reveals some structure of the  $L$  matrix. From the viewpoint of banking activities the sectoral organization is not particularly relevant any more. The activities of the sectors differ only slightly and only a few banks are specialized in specific lines of business. The 881 independent banks in our sample are to the largest extent universal banks.

### 3.2 Credit Exposure Data

We can get a rough breakdown for the banks' loan portfolio to non-banks by making use of the major loans register of OeNB (GKE). This database contains all loans exceeding a volume of 364,000 Euro. For each bank we use the amount and number of corporate loans, which are classified into 59 industry sectors according to the NACE standard plus 3 aggregate foreign non bank sectors grouped by industrialized countries, non industrialized countries, and Eastern Europe.<sup>7</sup> Since only loans above a volume threshold are reported we have to introduce domestic and foreign residual categories calculated from the difference between the total loan volume numbers in the banks balance sheets and the volume numbers of the major loan register.

Combining this information with data from the Austrian rating agency Kreditschutzverband von 1870 (KSV) we can estimate the riskiness of a loan in a certain industry. The KSV database gives us time series of default rates for the different NACE branches. From this statistics we estimate the average default frequency and its standard deviation for each NACE branch. These data serve as our input to the credit risk model.

For the part of loans we can not allocate to industry sectors we have no default statistics and no numbers of loans. To construct insolvency statistics for the residual sector we take averages from the data that are available. To construct a number of loans figure for the residual sector we assume the share of loan numbers in industry and in the residual sector is proportional to the share of loan volume between these sectors. We

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feisen banks, Volksbanken, building associations, savings and loan associations, and special purpose banks.

<sup>7</sup>Because of the close economic links between Austria and Central and Eastern European Countries we chose to define them as own category in the loan portfolio

should note that the insolvency series is very short. The series are available semi-annually beginning with January 1997.

### 3.3 Market Data

Some positions on the banks' asset portfolios are subject to market risk. We collect daily market data corresponding to the exposure categories over twelve years from September 1989 to September 2002 from Datastream. These data are used for the creation of scenarios. Specifically we collect exchange rates of USD, JPY, GBP and CHF to the Euro (Austrian Schilling before 1999) to compute exchange rate risk. As we only have data on domestic and international equity exposure we include the Austrian index ATX and the MSCI-world index in our analysis. To account for interest rate risk, we compute zero bond prices for maturities of three month, one, five and ten years, using zero rates in EUR, USD, JPY, GBP and CHF.<sup>8</sup>

## 4 Estimating Interbank Liabilities from Partial Information

If we want to apply the network model to our data we have the problem that they contain only partial information about the inter-bank liability matrix  $L$ . The bank by bank record of assets and liabilities with other banks gives us the column and row *sums* of the matrix  $L$ . Furthermore we know some structural information. For instance we know that the diagonal of  $L$  must contain only zeros since banks do not have claims and liabilities against themselves. We furthermore exploit the sectoral structure of the Austrian banking system to determine many of the entries in  $L$  *exactly*. In this way we can pin down 72% of all entries of the matrix  $L$ . Therefore by exploiting the sectoral information, we actually know a major part of  $L$  from our data.

We *estimate* the remaining 28% of the entries of  $L$  by optimally exploiting the information we have. Our ignorance about the unknown parts of the matrix should be reflected in the fact that all these entries are treated uniformly in the reconstruction process. The procedure should be furthermore adaptable to include any new information that

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<sup>8</sup>Sometimes a zero bond series is not available for the length of the period we need for our exercise. In these cases we took swap rates.

might get available in the process of data collection. In the following we use a procedure that formulates the reconstruction of the unknown parts of the  $L$  matrix as an *entropy optimization problem*.

What this procedure does can intuitively be explained as follows: It finds a matrix that treats all entries of the matrix as balanced as possible and that fulfils all the constraints we know of. This can be formulated as minimizing a suitable measure of distance between the estimated matrix and a matrix that reflects our a priori knowledge on large parts of bilateral exposures. It turns out that the so called *cross entropy* measure is a suitable concept for this task (see Fang, Rajasekra, and Tsao (1997) or Blien and Graef (1997)). For a formal description we refer to Appendix B.

Due to data inconsistencies the application of entropy optimization is not straightforward. For instance the liabilities of all banks in sector  $k$  against all banks in sector  $l$  do typically not equal the claims of all banks in sector  $l$  against all banks in sector  $k$ .<sup>9</sup> We propose a workaround to this problem which is described in detail in Appendix B.

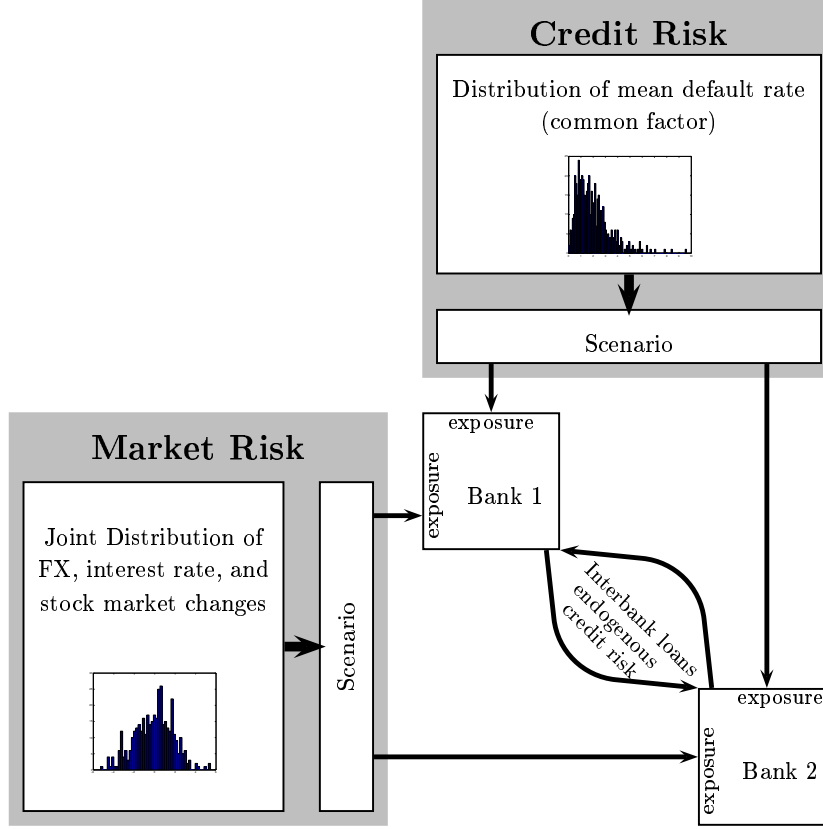
We see two main advantages of this method to deal with the incomplete information problem raised by our data. First the method is fairly flexible with respect to the inclusion of additional information we might get from different sources. Second, there exist computational procedures that are easy to implement and that can deal efficiently with very large problems (see Fang, Rajasekra, and Tsao (1997)). Thus problems similar to ours can be solved efficiently and quickly on an ordinary personal computer, even for very large banking systems.

## 5 Creating Scenarios

Our model of the banking sector uses different scenarios to model uncertainty. In each scenario banks face gains and losses from FX and interest rate changes as well as from equity price changes and losses from loans to non-banks. Some banks may fail. This possibly causes subsequent failures of other banks, as it is modeled in our network clearing framework. Hence the credit risk in the inter-bank network is modeled endogenously while all other risks are reflected in the position  $e_i$ . The perspective taken in our analysis is to ask, what are the consequences of different scenarios for  $e_i$  on the whole banking system.

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<sup>9</sup>Some of the inconsistencies seem to suggest that the banks assign some of their counterparties to the wrong sectors.



**Figure 1.** The figure shows the basic structure of the model. Banks are exposed to shocks from credit risk and market risk according to their respective exposures. Due to these economic shocks some banks may default. Inter-bank credit risk is endogenously explained by the network model. The clearing of the inter-bank market determines the solvency of other banks and defines endogenous default probabilities for banks as well as the respective recovery rates.

Figure 1 shows the basic structure of our model.

We choose a standard risk management framework to model the shocks to banks. To simulate scenario losses that are due to exposures to market risk we conduct a historical simulation where we expose all banks simultaneously to the same realizations of the risk factors. To capture losses from loans to non-banks we use a credit risk model.

Table 1 shows, which balance sheet items are included in our analysis and how the risk exposure is modeled. Market risk (stock price changes, interest rate movements and FX rate shifts) are captured by a historical simulation approach (HS) for all items except other assets and other liabilities, which includes long term major equity stakes in not-listed companies, non financial assets like property and IT-equipment and cash on the

<b>Assets</b>	Interest rate/ stock price risk	Credit risk	FX risk
short term government bonds and receivables	Yes (HS)	No	Yes (HS)
loans to other banks	Yes (HS)	endogenous by clearing	Yes (HS)
loans to non banks	Yes (HS)	credit risk model	Yes (HS)
bonds	Yes (HS)	no as mostly government	Yes (HS)
stock holdings	Yes (HS)	No	Yes (HS)
other assets	No	No	No
<b>Liabilities</b>			
liabilities other banks	Yes (HS)	endogenous by clearing	Yes (HS)
liabilities non banks	Yes (HS)	No	Yes (HS)
securitized liabilities	Yes (HS)	No	Yes (HS)
other liabilities	No	No	No

**Table 1.** The table shows how risk of the different balance sheet positions is covered in our scenarios. HS is a shortcut for historic simulation.

asset side and equity capital and provisions on the liability side. Credit losses from non-banks are modeled via a credit risk model. The credit risk from bonds is not included since most banks hold only government bonds. The credit risk in the inter-bank market is determined endogenously.

## 5.1 Market Risk: Historical Simulation

We use a historical simulation approach as it is documented in the standard risk management literature (Jorion (2000)) to assess the market risk of the banks in our system. This methodology has the advantage that we do not have to specify a certain parametric distribution for our returns. Instead we can use the empirical distribution of past observed returns and thus capture also extreme changes in market risk factors. By this procedure we capture the joint distribution of the market risk factors and thus take correlation structures between interest rates, stock markets and FX markets into account.

To estimate shocks on bank capital stemming from market risk, we include positions in foreign currency, equity, and interest rate sensitive instruments. For each bank we collect foreign exchange exposures for USD, JPY, GBP, and CHF only as no bank in our sample has open positions of more than 1% of total assets in any other currency. From the MAUS database we get exposures to foreign and domestic stocks, which is equal to

the market value of the net position held in these categories. The exposure to interest rate risk can not be read directly from the banks' monthly reports. We have information on net positions in all currencies combined for different maturity buckets (up to 3 month but not callable, 3 month to 1 year, 1 to 5 years, more than 5 years). These given maturity bands allow only a quite coarse assessment of interest rate risk.

Nevertheless the available data allow us to estimate the impact of changes in the term structure of interest rates. To get an interest rate exposure for each of the five currencies EUR, USD, JPY, GBP and CHF we split the aggregate exposure according to the relative weight of foreign currency assets in total assets. This procedure gives us a vector of 26 exposures, 4 FX, 2 equity, and 20 interest rate, for each bank. Thus we get a  $N \times 26$  matrix of market risk exposure.

We collect daily market prices over 3,220 trading days for the risk factors as described in subsection 3.3. From the daily prices of the 26 risk factors we compute daily returns. We re-scale these to monthly returns assuming twenty trading days and construct a  $26 \times 3219$  matrix  $R$  of monthly returns.

For the historical simulation we draw 10,000 scenarios from the empirical distribution of returns. To illustrate the procedure let  $R_s$  be one such scenario, i.e. a column vector from the matrix  $R$ . Then the profits and losses that arise from a change in the risk factors as specified by the scenario are simply given by multiplying them with the respective exposures. Let the exposures that are directly affected by the risk factors in the historical simulation be denoted by  $a$ . The vector  $aR_s$  contains then the profits or losses each bank realizes under the scenario  $s$ . Repeating the procedure for all 10,000 scenarios, we get a distribution of profits and losses due to market risk.

## 5.2 Credit Risk: Calculating Loan Loss Distributions

For the modeling of loan losses we employ one of the standard modern credit risk models, CreditRisk+.<sup>10</sup> While CreditRisk+ is designed to deal with a single loan portfolio we have to deal with a *system* of portfolios since we have to consider all banks simultaneously. For the purpose of our analysis the correlation between loan losses across banks is important.

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<sup>10</sup>A recent overview on different standard approaches to model credit risk is Crouhy, Galai, and Mark (2000). CreditRisk+ is a trademark of Credit Suisse Financial Products (CSFP). It is described in detail in CSFP Credit Suisse (1997)

The adaptation of the model to deal with such a system of loan portfolios turns out to be straightforward.

The loan loss distribution in the CreditRisk+ model is driven by two sources of uncertainty. First, economic uncertainty affects all loans. This models business cycle effects on average industry defaults. The idea is that default frequencies increase in a recession and decrease in booms. Second, for a given economic shock defaults are assumed to be conditionally independent. The conditional loss distribution can be derived analytically using an iterative algorithm. Assumptions about recovery rates can be imposed at various sophistication levels. For our purposes it is sufficient to work with a very simple assumption of a fixed recovery rate. Throughout our calculations we assume a recovery rate of corporate loans of 50%

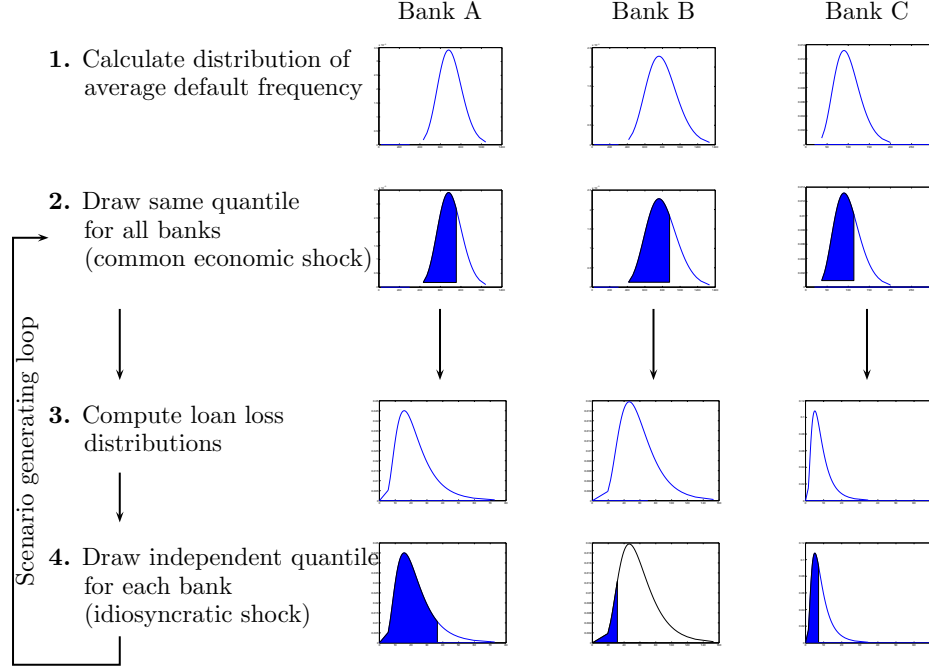
We construct the bank loan portfolios by decomposing the bank balance sheet information on loans to non banks into volume and number of loans in different industry sectors according to the information from the major loan register. The rest is summarized in a residual position as described in Section 3. Using the KSV insolvency statistics for each of the 59 industry branches and the three foreign sectors and the proxy insolvency statistics for the residual sectors, we can assign an unconditional expected default frequency and a standard deviation of this frequency to each loan. In line with the CreditRisk+ specification, we aggregate these numbers for each bank. The unconditional expected default frequency gives us information on the risk of the bank's loan portfolio. The standard deviation indicates how much default rates may vary over time and thus model the exposure to the economic shock.

Figure 2 illustrates the procedure for scenario generation in our extended CreditRisk+ framework. We follow a four step procedure to generate scenarios for the whole banking system.<sup>11</sup> In step one of the simulation we compute the distribution of each bank's average default frequencies.<sup>12</sup> Then we draw for each bank a realization from the bank's individual distribution of average default frequencies. To model this as an economy wide shock, we draw the same quantile for all banks in the banking system (step 2). Given this realization of the average default frequency, defaults are assumed to be conditionally independent. We can then calculate a conditional loss distribution for each bank (step 3). Finally (step

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<sup>11</sup>For a single loan portfolio these four stages can be combined and the unconditional loss distribution can be derived. Our model is identical to the CreditRisk+ model in this case.

<sup>12</sup>In CreditRisk+ the distribution of the expected default frequency is specified as a gamma distribution. The parameters of the gamma distribution can be determined by the average number of defaults in the loan portfolio and its standard deviation.



**Figure 2.** Computation of Credit loss scenarios following an extended CreditRisk+ model. Based on the composition of the individual bank's loan portfolio we estimate the distribution of the mean default rate for each bank (step 1). Reflecting the idea of a common economic shock we draw the same quantile from each bank's mean default rate distribution (step 2). Conditional on this draw, we can compute each bank's individual loan loss distribution (step 3). The scenario loan losses are then drawn independently for each bank to reflect an idiosyncratic shock (step 4). 10,000 scenarios are drawn repeating steps 2 to 4.

4) we draw loan losses.<sup>13</sup>

### 5.3 Combining Market Risk, Credit Risk, and the Network Model

The credit losses across scenarios are combined with the results of the historic simulations to create  $e_i$  for each bank. By the network model the inter-bank payments for each scenario  $e_i$  are then calculated (see Figure 1). Thus we get endogenously a distribution of clearing vectors, default frequencies, recovery rates and statistics on the decomposition

<sup>13</sup>We apply standard variance reduction techniques in our Monte Carlo simulation. We go through the quantiles of the distribution of average default frequencies at a step length of 0.01. Thus, we draw hundred economy wide shocks from each of which we draw 100 loan loss scenarios, yielding a total number of 10,000 scenarios.

into fundamental and contagious defaults.

## 6 Results

The network model generates a *distribution* of clearing vectors  $p^*$  and therefore also a distribution of insolvencies for each individual bank across scenarios: Whenever a component in  $p^*$  is smaller than the corresponding component in  $d$  the bank has not been able to honor its inter-bank promises. The relative frequency of default across scenarios is then interpreted as a *default probability*.

To discuss the effects of risk scenarios on the banking system it is useful to impose additional assumptions reflecting the time horizon we have in mind. Although technically the network model works with a fixed future date at which all claims are cleared simultaneously we can model time horizons by imposing assumptions on details of the clearing process. We can model a *short run* perspective by assuming that there will be no inter-bank payments after netting following a bank default. In contrast we model a *long run* perspective by assuming that the residual value of an insolvent bank can be fully transferred to the creditor institutions up to some bankruptcy costs according to the rules of the clearing mechanism.

Clearly both situations are of interest for a regulator. The short run assumptions help to estimate the amounts that might have to be ready immediately for emergency crises intervention to prevent wide spread contagion of defaults. The long run assumptions can then give an estimate of the eventual costs of a shock to the system. Against these hypothetical situations crises intervention decisions can be evaluated.

### 6.1 Frequency of Default

Panel A of Table 2 shows the default probabilities for the short run. We see the 10 and 90 percent quantiles as well as the median of the distribution of individual bank default probabilities grouped by banks size. The last line shows these numbers for the entire banking system. Banks are sorted into three groups by the size of total assets. Banks that are defined as small if they are below the first quartile of the total assets distribution. Banks between the first quartile and the 90% decile are defined as medium whereas banks

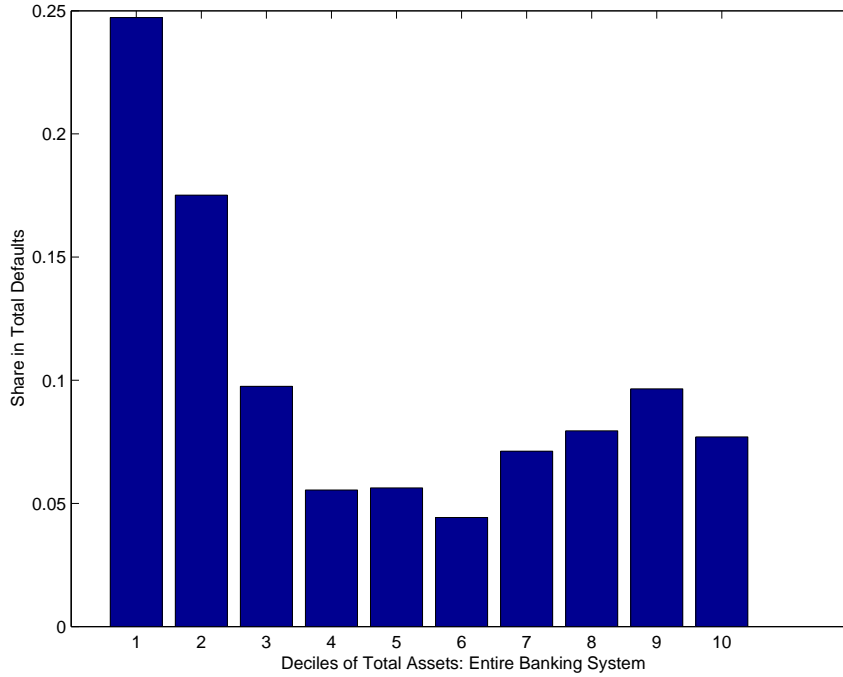
	Panel A: Short Run Scenario			Panel B: Long Run Scenario		
	10%-Quant.	Median	90%-Quant.	10%-Quant.	Median	90%-Quant.
Small	0.00%	0.44%	4.99%	0.00%	0.40%	4.92%
Medium	0.00%	0.12%	1.98%	0.00%	0.06%	1.66%
Large	0.00%	0.07%	1.31%	0.00%	0.00%	0.09%
All banks	0.00%	0.16%	2.91%	0.00%	0.10%	2.60%

**Table 2.** Default probabilities of individual banks in the short run and long run, grouped by quantiles of size of total assets and for the entire banking system. Small banks are defined to be in the first quartile of the total asset distribution; medium banks are defined as banks between the lower quartile and the 90% quantile of the total asset distribution. Large banks are defined as institutions in the top decile of the total asset distribution. The short run analysis is under the assumption that insolvent banks pay nothing in the inter-bank market; in the long run the remaining value of the bank is proportionally shared among claimants assuming zero bankruptcy costs.

above the 90% decile are defined as large.

The banking system’s overall stability as measured by the median default probability across the entire banking system is both in the short run and in the long run satisfactory. The median default probability is way below one percent in both cases. In the short run (Panel A of Table 2) the median number of failure scenarios measured across all the banks in the system is 16 out of 10,000. In the long run payments can be made above the payments that come from the netting of claims. If the value of an insolvent institution can be transferred to the creditor banks clearly the distribution of default probabilities across banks changes. The results are reported in Panel B of Table 2. We see that the default probability for small and medium sized banks decreases in the long run scenario only slightly. The mean default probability (not shown in the table) in the long run is 0.8%. The picture is different for large banks where the decrease is rather pronounced. This finding remains more or less unchanged if one introduces the assumption that a certain amount of resources is lost during a bankruptcy procedure. For instance James (1991) estimates these costs at 10% of total assets. With this assumption the median default probability in the long run rises to 0.12% for the entire banking system. It raises the median default probability of medium sized banks to 0.08% and leaves it unchanged for the small banks.

The default risk seems to decrease with bank size. This picture can be refined if we group banks by deciles of the size distribution of total assets and plot the share of banks in these deciles in the number of total defaults across banks and scenarios (Figure 3



**Figure 3.** The figure shows the share in total defaults of banks in each decile of the total asset distribution (long run case). Small banks account for a large fraction of total defaults. Banks in the fourth, fifth, and sixth decile account for a less than proportional share of defaults.

for the long run case). The relation between bank size and default probability is now more ambiguous. The default probability is high for small banks (the lowest three deciles account for approximately 50% of all defaults). It decreases sharply for the next three deciles and then increases again.

## 6.2 Recovery Rates

The model allows us to give an estimate of the value recoverable from defaulted counterparties in the inter-bank market. This information is interesting in its own right because relatively little is known about recoveries from inter-bank credits. This estimate is of course only relevant for the long run since our assumptions imply that the recovery rate in the short run is zero apart from netting.

The following numbers refer to the long run assumption that the residual value of an insolvent institution is fully transferred to the creditors. In each scenario where bank  $i$  defaults a recovery rate is calculated by dividing  $p_i^*$  - the amount paid by bank  $i$  under

Size	10%-Quantile	Median	90%-Quantile
Small	0.00%	23.02%	81.06%
Medium	20.17%	76.36%	95.96%
Large	10.86%	89.84%	98.16%
Banking system	0.00%	65.83%	95.54%

**Table 3.** Recovery rates from inter-bank exposures in the long run grouped by quantiles of size of total assets and for the entire banking system. Small Banks are defined to be institutions in the first quartile of the total asset distribution; large banks are defined as institutions in the top decile of the total asset distribution.

the clearing mechanism - by  $d_i$  - the amount initially promised by bank  $i$ . These rates for bank  $i$  are averaged across scenarios where bank  $i$  defaults. The values of recovery rates are reported in Table 3.<sup>14</sup>

These recovery rates are implied by the network model. In practice recovery rates might be higher because some of the exposures will be collateralized. We have included no assumptions about collateral since we have no appropriate information. What is remarkable in these numbers is that the median recovery rate for the large institutions is high. This indicates that if they fail, the losses for the counterparties will be moderate. The median recovery rate taken over the entire banking system is 66%. It should also be noted that these recovery rates drop sharply if we deduct bankruptcy costs of 10% of total assets as estimated by James (1991). In this case the median recovery rate goes down to zero and reaches a value of about 46% in the 90% quantile of the recovery rate distribution across the entire banking system.

### 6.3 Systemic stability: Fundamental versus Contagious Defaults

Let us now turn to the decomposition into fundamental and contagious defaults. This decomposition is particularly interesting from the viewpoint of systemic stability. It is also interesting to get an idea about contagion of bank defaults from a real dataset since the empirical importance of domino effects in banking has been controversial in the literature.<sup>15</sup> Bank defaults may be driven by large exposures to market and credit risk or from an inadequate equity base (fundamental default). Bank defaults may however also be initiated by contagion: as a consequence of a chain reaction of other bank failures in the

<sup>14</sup>To calculate the recovery rates we clear the system without netting. In the case of clearing with netting we could clearly not simply calculate recovery rates as  $p_i^*/d_i$ .

<sup>15</sup>See in particular the detailed discussion in Kaufman (1994).

	Short Run			Long Run		
	Fundamental	Contagious	Total	Fundamental	Contagious	Total
0-10	87.03%	1.15%	88.18%	88.17%	0.01%	88.18%
11-20	3.09%	1.36%	4.45%	4.43%	0.02%	4.45%
21-30	1.54%	0.69%	2.23%	2.20%	0.03%	2.23%
31-40	0.80%	0.66%	1.46%	1.36%	0.10%	1.46%
41-50	0.33%	0.63%	0.96%	0.90%	0.06%	0.96%
51-60	0.20%	0.42%	0.62%	0.52%	0.10%	0.62%
61-70	0.09%	0.33%	0.42%	0.36%	0.06%	0.42%
71-80	0.04%	0.29%	0.33%	0.20%	0.13%	0.33%
81-90	0.02%	0.32%	0.34%	0.19%	0.15%	0.34%
91-100	0.00%	0.19%	0.19%	0.11%	0.08%	0.19%
more	0.07%	0.75%	0.82%	0.21%	0.61%	0.82%
Total	93.21%	6.79%	100.00%	98.65%	1.35%	100.00%

**Table 4.** Probabilities of fundamental and contagious defaults in the short run and in the long run. A fundamental default is due to the losses arising from exposures to market risk and credit risk to the corporate sector, while a contagious default is triggered by the default of another bank that cannot fulfill its promises in the inter-bank market. The short run analysis is under the assumption that insolvent banks pay nothing in the inter-bank market, whereas zero bankruptcy costs are assumed in the long run scenario. Banks are grouped by fundamental defaults. The probability that only fundamental defaults occur is shown as well as the probability that fundamental and contagious defaults are observed.

system (contagious default). These risks are not visible by a regulatory setup that focuses on individual banks' "soundness". Table 4 summarizes the probabilities of fundamental and contagious defaults in our data.

We list the probabilities of 0 – 10 banks defaulting fundamentally and the probability of banks defaulting contagiously as a consequence of this event. The next row shows similar information for the event that 11 – 20 banks get insolvent and the probability that other banks default contagiously as a consequence of this etc. We see that in the short run the incidence of contagion can become non-negligible. Of all default scenarios almost 7% can be classified as scenarios with contagious defaults.

The incidence of contagious default in the long run scenario roughly decreases by a factor of 5. This shows that the long run picture consists predominantly of scenarios with fundamental defaults and that domino effects play not a particularly prominent role. However we have seen that in the short run the incidence of contagion can reach fairly high levels and might give reasons for concern. We can therefore not conclude from our analysis that the incidence of contagious default is a quantity that might be safely ignored for all practical purposes.

	Panel A: Short Run Scenario			Panel B: Long Run Scenario		
	10%-Quant.	Median	90%-Quant.	10%-Quant.	Median	90%-Quant.
Small	0.00%	0.03%	0.15%	0.00%	0.00%	0.03%
Medium	0.00%	0.03%	0.12%	0.00%	0.00%	0.01%
Large	0.00%	0.02%	0.26%	0.00%	0.00%	0.04%
All banks	0.00%	0.03%	0.13%	0.00%	0.00%	0.01%

**Table 5.** Probabilities of contagious defaults, i.e., defaults because other banks do not fully honor their promises. Panel A is under the assumption that defaulted banks pay nothing in the inter-bank market, whereas zero bankruptcy costs are assumed in Panel B. Large and small banks are in the top decile and the lowest quartile of the total asset distribution, respectively.

We can also directly calculate the probability of contagious defaults along the lines of Table 2. The numbers are given in Table 5. The probability of a contagious default of an individual bank in the long run is remarkably low (90% of the banks face a contagious default in at most 1 scenario out of 10,000). The mean contagious default probability (not shown in the table) is only 0.0062%. In the short run this probability is slightly higher. However contrary to the fact that the total default probability is lower for large banks than it is for small sized banks we find that large banks face more contagious defaults than small banks do.

Given the number of total defaults several percentiles of the ratio of contagious defaults to total defaults are given in Table 6. In the short run case contagious defaults can account for up to 3/4 of all defaults, i.e. there exists a scenario where 3/4 of the defaults are contagious. Although this is not a representative scenario (the median of the fraction is in a range of 0% to 14% depending on the number of total defaults) it is important in terms of the stability of the system. It indicates that despite the fact that contagion is a rare event there are situations where it accounts for a major part of defaults. The results in Panel A of Table 6 confirm the intuition that the fraction of contagious defaults increases in the number of total defaults. As we have already seen before contagion is a minor problem in the long run case (Panel B of Table 6).

## 6.4 Estimating the Costs of Crises Intervention

We can use the model to estimate the costs of crises intervention by asking how much funds would have to be available to avoid contagious defaults or even fundamental defaults. These numbers can be estimated by using the network model. In the case of

fundamental defaults we have to calculate the amounts needed to make the value of each bank non negative across scenarios in the first round of the fictitious default algorithm. For contagious default we have to do the same calculation for rounds beyond round zero. The aggregate amount of these funds can be analyzed by looking at the percentiles of the intervention cost distribution across scenarios calculated in the way. Table 7 reports our results for the short run.

What is remarkable here is that the amounts that have to be ready to prevent only contagious defaults are not very big. For the data set analyzed here the amount is roughly 50 million Euro. This is roughly 0.1% of the banking system's total assets. We have seen before that the share of scenarios where no contagious defaults occur is more than 90%. Thus the fundamental defaults play a more prominent role. A fundamental default may give rise to concerns from the side of the regulator as well. If we would also ask how much we should stand ready to inject into the system to avoid fundamental defaults we get number that are larger by an order of 100 or more. In the long run the costs to avoid contagious defaults decline sharply.

## 7 Discussion

### 7.1 Contagion and Bankruptcy Costs

In the preceding analysis we have seen that contagious defaults turned out to differ quite substantially between the short run and the long run simulation. We have distinguished both simulations basically by assumptions on possible recoveries from defaulted counterparties. This raises the question on how contagion depends exactly on the amount of bankruptcy costs. In the following we use our data to find the function that maps bankruptcy costs measured as percent of the value of total assets into contagious defaults.

Figure 4 shows the impact of bankruptcy costs on contagious defaults. For each level of bankruptcy costs we compute the distribution of contagious defaults across scenarios. The graph shows the tail of the distribution, i.e., how many banks fail in the bad scenarios. Efficient bankruptcy resolution is crucial to limit contagion. We see little contagion for low bankruptcy costs and very high contagion for levels above 30%. The jump in the maximum number of contagious defaults clearly shows that financial stability can be

Panel A: Short Run Scenario:

	Prob. of State	Minimum	10%-Quantile	Median	90%-Quantile	Maximum
0 to 10	87.14%	0.00%	0.00%	0.00%	0.00%	50.00%
11 to 20	4.80%	0.00%	0.00%	0.00%	9.09%	20.00%
21 to 30	1.91%	0.00%	0.00%	0.00%	3.70%	65.38%
31 to 40	1.60%	0.00%	0.00%	2.63%	44.12%	63.33%
41 to 50	1.45%	0.00%	0.00%	2.44%	32.50%	40.00%
51 to 60	0.63%	0.00%	0.00%	1.96%	25.09%	30.77%
61 to 70	0.55%	0.00%	0.00%	2.99%	21.54%	25.37%
71 to 80	0.27%	0.00%	0.00%	13.51%	22.54%	23.61%
81 to 90	0.25%	0.00%	1.16%	11.11%	17.24%	20.48%
91 to 100	0.15%	1.02%	1.09%	4.35%	15.46%	16.84%
More	1.25%	0.00%	1.17%	13.33%	41.54%	73.30%
All banks	100.00%	0.00%	0.00%	0.00%	0.00%	73.30%

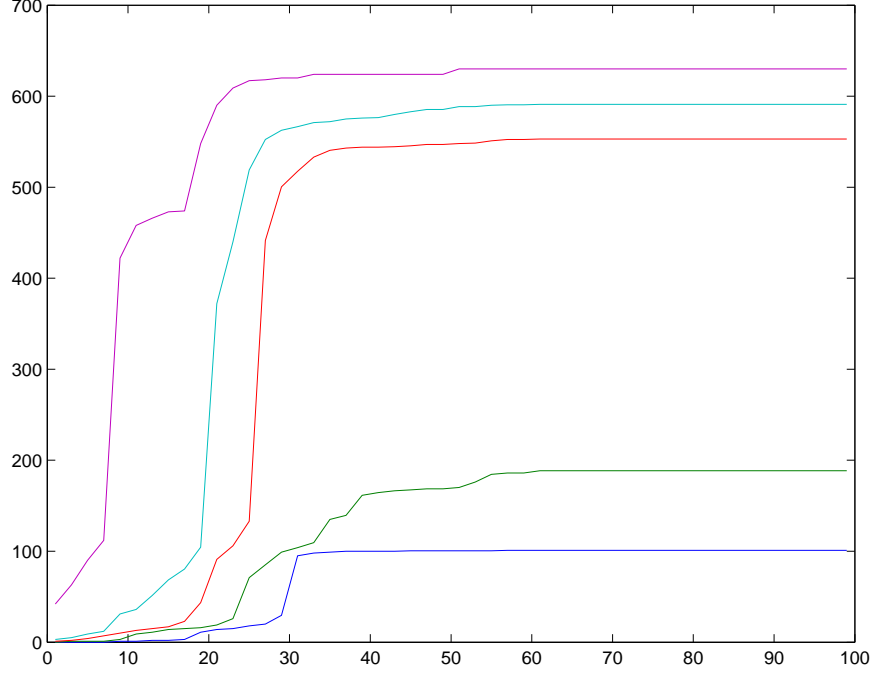
Panel B: Long Run Scenario

	Prob. of State	Minimum	10%-Quantile	Median	90%-Quantile	Maximum
0 to 10	87.40%	0.00%	0.00%	0.00%	0.00%	12.50%
11 to 20	4.87%	0.00%	0.00%	0.00%	0.00%	7.69%
21 to 30	2.44%	0.00%	0.00%	0.00%	0.00%	3.57%
31 to 40	1.44%	0.00%	0.00%	0.00%	0.00%	3.13%
41 to 50	1.04%	0.00%	0.00%	0.00%	0.00%	4.08%
51 to 60	0.67%	0.00%	0.00%	0.00%	1.77%	2.00%
61 to 70	0.42%	0.00%	0.00%	0.00%	1.54%	1.67%
71 to 80	0.33%	0.00%	0.00%	0.00%	1.37%	2.82%
81 to 90	0.34%	0.00%	0.00%	0.00%	2.36%	3.53%
91 to 100	0.22%	0.00%	0.00%	0.00%	2.16%	3.03%
More	0.83%	0.00%	0.00%	1.57%	7.53%	10.07%
All banks	100.00%	0.00%	0.00%	0.00%	0.00%	12.50%

**Table 6.** Probability of bank defaults and amount of contagion. Scenarios are grouped by total bank defaults (fundamental and contagious). For each group the probability of that state is shown as well as quantiles of the fraction of contagious defaults. In the long run analysis we assume zero bankruptcy costs, whereas in the short term scenario insolvent banks are assumed to default completely and pay nothing.

Quantiles	90%	95%	99%	100%
Fundamental Default	61.82	194.85	895.5133	5666.89
Contagious Default Short Run	0	0	1.83	46.44
Contagious Default Long Run	0	0	0.12	20.01

**Table 7.** Costs of avoiding fundamental and contagious defaults: In the first row we give estimates for the 90, 95, 99 and 100 percentile of the avoidance cost distribution across scenarios for fundamental defaults. The second row shows the amounts necessary to avert contagious defaults once fundamental defaults have occurred. Costs are in million Euro.



**Figure 4.** Number of contagious defaults and bankruptcy costs. The graphs show the maximum number of contagious defaults as well as the 99.5%, 99%, 98%, and 97% quantiles of the distribution for different bankruptcy costs. Bankruptcy costs are defined as percentage of total assets lost in case of bankruptcy.

enhanced when bankruptcy costs are kept very low. This highlights the importance of a lender of last resort and an efficient crisis resolution policy. When regulators are able to support efficient bankruptcy resolutions they can effectively limit contagion in a crisis scenario.

## 7.2 The Role of the Network Structure

In the theoretical literature on contagion and banking crises Allen and Gale (2000) have suggested that the pattern of linkages in the inter-bank market is critical for financial fragility. In particular they contrast two kinds of inter-bank market structures, which are called *complete* and *incomplete*. A complete structure refers to a network topology of the inter-bank market where all banks are connected with each other by claims or liabilities. An incomplete structure is one in which banks are only partially connected. Allen and Gale (2000) study an example of an incomplete structure where banks hold inter-bank deposits among each other. In a liquidity insurance framework they analyze

the risk allocation and fragility properties of a banking system with four banks which face different liquidity shocks but can enter risk sharing agreements to achieve improved allocations of liquidity contrary to a situation without an inter-bank market. The authors show that the fact whether the inter-bank market structure is complete or incomplete is decisive whether an aggregate liquidity shock leads to contagion and financial crises or not. They conclude that a complete structure is more robust than an incomplete structure. A complete structure leads to a better distribution of the risk of an aggregate liquidity shock and does therefore not create a necessity of costly liquidation of long term assets. An incomplete structure may however lead to large effects of an aggregate liquidity shock because banks can only turn on their neighboring region for liquidity and may enforce premature liquidation with a knock on effect to other regions.

The analysis in Allen and Gale (2000) raises the question whether a complete inter-bank market structure can in general make a banking system more resilient to aggregate shocks. We can use our framework to provide some empirical evidence. If we construct the inter-bank matrix  $L$  by just imposing the row sum and column sum constraints on our data and use a prior liability matrix that gives equal weight to all matrix entries except the diagonal (where entries are zero) the entropy maximization procedure will induce exactly a complete structure as suggested by Allen and Gale (2000). The initial structure which exploits the structural information about the multi-tier architecture of the Austrian banking system can be characterized as an incomplete market structure.

The simulation results with the complete structure and the long run scenario are reported in Table 8. Analogous to Table 4, we group bank failure scenarios by the number of fundamental defaults. For each group, we compute the probability that only fundamental and that fundamental and contagious defaults occur.

What is striking here is that contrary to the example discussed in Allen and Gale (2000) the complete market structure in our case leads to an *increase* in scenarios with contagious defaults by roughly three percentage points. We think that the conclusion we can draw from this exercise is that the classification into complete and incomplete structures as suggested by Allen and Gale (2000) does not give the whole story if we want to understand the role played by the network topology of inter-bank linkages for financial fragility of a banking system.

	Fundamental	Contagious	Total
0-10	87.93%	0.25%	88.18%
11-20	4.22%	0.23%	4.45%
21-30	1.68%	0.55%	2.23%
31-40	0.95%	0.51%	1.46%
41-50	0.60%	0.36%	0.96%
51-60	0.28%	0.34%	0.62%
61-70	0.07%	0.35%	0.42%
71-80	0.03%	0.30%	0.33%
81-90	0.01%	0.33%	0.34%
91-100	0.00%	0.19%	0.19%
more	0.03%	0.79%	0.82%
Total	95.80%	4.20%	100.00%

**Table 8.** Probabilities of fundamental and contagious defaults assuming zero bankruptcy costs and a complete market structure, i.e. banks diversify their inter-bank business as much as possible. Bank failure scenarios are grouped by the number of fundamental defaults. For each group, the probability that only fundamental and that fundamental and contagious defaults occur are shown. A fundamental default is due to the losses arising from exposures to market risk and credit risk to the corporate sector, while a contagious default is triggered by the default of another bank who cannot fulfill its promises in the inter-bank market.

## 8 Conclusions

In this paper we have developed a new framework for the risk assessment of a banking system. The innovation is that we judge risk at the level of the entire banking system rather than at the level of an individual institution.

Conceptually it is possible to take this perspective by a systematic analysis of the impact of a set of macroeconomic risk factors on banks in combination with a network model of mutual credit relations. To be able to employ the framework empirically it is designed in a way such that the data required as input is available at the regulatory authority. This is exactly the institution for which an assessment method like the one suggested here is of crucial interest.

Since our method is a first step, there are certainly many issues that have to be further discussed to get a firm judgment how much we can trust assessments generated with the help of our model. We want to point out at this stage, what we see as the main advantages of our general approach. First the system perspective can uncover exposures to aggregate risk that are invisible for banking supervision relying on the assessment of single institutions only. It disentangles the risk that comes from fundamental shocks from

the risks that come from the failure of other banks. As we gain experience with the model for more cases and maybe also for other banking systems this might create a possibility to qualify the actual importance of contagion effects that have received so much attention in the theoretical debate. Second we think that our framework can redirect the discussion about systemic risk in banking from continuous refinements and extensions of capital adequacy regulation for individual banks to the crucial issue of how much risk is actually borne by the banking system. In this discussion the framework might be useful to get a clearer picture of actual risk exposure because it allows for thought experiments. Third the model does not rely on a sophisticated theory of economic behavior. In fact the model is really a tool to read data in a particular way. All it does is to make the *consequences* of a given liability and asset structure in combination with realistic shock scenarios *visible* in terms of implied technical insolvencies of institutions. We think that in this context this feature is a definitive advantage because it makes it easier to validate the model. Fourth the model is designed to exploit *existing* data sources. Although these sources are not ideal our approach shows that we can start to think about financial stability at the system level with existing data already.

We hope that our ideas will turn out useful for regulators and central bankers by offering a practicable way to read the data they have at their very doorsteps in the light of aggregate risk exposure of the banking system. We therefore hope to have given a perspective of how a 'macroprudential' approach to banking supervision could proceed. We also do hope, however, that our paper turns out to be interesting for theoretical work in financial stability and banking as well and that the questions it raises will contribute in a fruitful way to the debate about the system approach to banking supervision and risk assessment.

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## A A Toy Example

Let us illustrate the concepts introduced in section 2 by an example: Consider a system with three banks. The inter-bank liability structure is described by the matrix

$$L = \begin{pmatrix} 0 & 0 & 2 \\ 3 & 0 & 1 \\ 3 & 1 & 0 \end{pmatrix} \quad (3)$$

Bank 3 has - for instance - liabilities of 3 with bank 1 and liabilities of 1 with bank 2. It has of course no liabilities with itself. The total inter-bank liabilities for each bank in the system is given by a vector  $d = (2, 4, 4)$ . With actual balance sheet data the components of the vector  $d$  correspond to the position *due to banks* for bank 1, 2 and 3 respectively. If we alternatively look at the column sum of  $L$  we get the position *due from banks*. Assume that we can summarize the net wealth of the banks that is generated from all other activities by a vector  $e = (1, 1, 1)$ . This vector corresponds to the difference of asset positions such as bonds, loans and stock holdings and liability positions such as deposits and securitized liabilities.

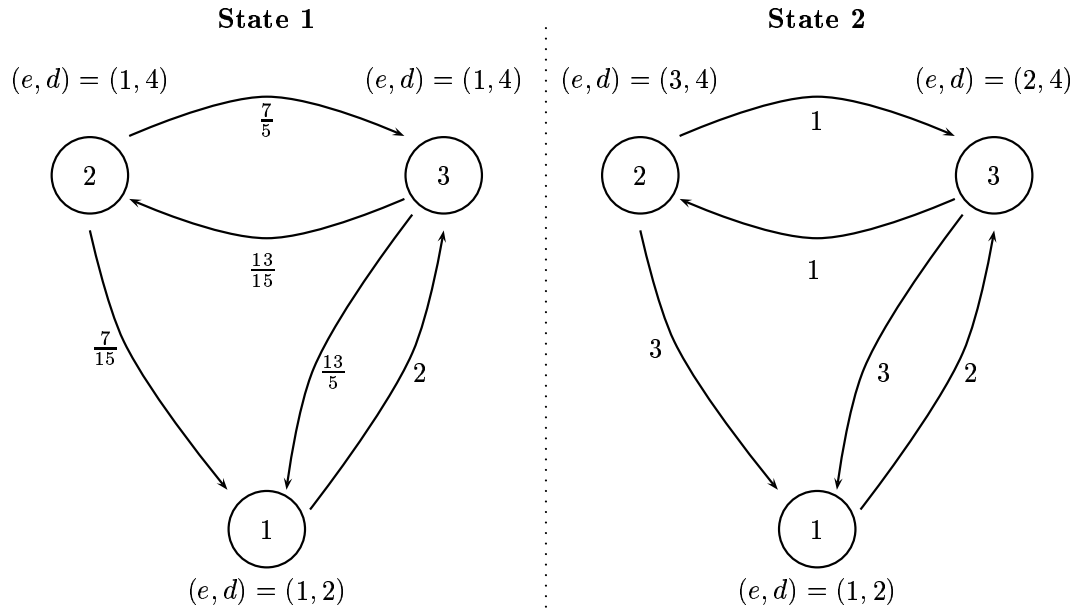
The normalized liability matrix  $\Pi$  is given by

$$\Pi = \begin{pmatrix} 0 & 0 & 1 \\ \frac{3}{4} & 0 & \frac{1}{4} \\ \frac{3}{4} & \frac{1}{4} & 0 \end{pmatrix}$$

Applying the fictitious default algorithm to the situation where  $e = (1, 1, 1)$  yields a clearing payment vector of  $p^* = (2, \frac{28}{15}, \frac{52}{15})$ . It is easy to check that bank 2 is fundamentally insolvent whereas bank 3 is “dragged into insolvency” by the default of bank 2.

Suppose  $e$  is uncertain whereas  $L$  is deterministic. Assume that we draw two realizations from the distribution of  $e$ . Let these two scenarios be the vectors  $e_1 = (1, 1, 1)$  and  $e_2 = (1, 3, 2)$ . Given the matrix  $L$ , in the first scenario banks 2 and 3 default whereas in the second scenario no bank is insolvent. The clearing payment vectors and the network structure for this example are illustrated in Figure 5.

The ex-ante the expected number of bank defaults, of fundamental and contagious defaults as well as expected average recovery rates from inter-bank credits are the averages over the scenarios.



**Figure 5.** Graphical representation of the simple toy-model network of inter-bank liabilities.

## B Estimating the $L$ matrix

Assume we have in total  $K$  constraints that include all constraints on row and column sums as well as on the value of particular entries. Let us write these constraints as

$$\sum_{i=1}^N \sum_{j=1}^N a_{kij} l_{ij} = b_k \quad (4)$$

for  $k = 1, \dots, K$  and  $a_{kij} \in \{0, 1\}$ .

We want to find the matrix  $L$  that has the least discrepancy to some a priori matrix  $U$  with respect to the (generalized) cross entropy measure

$$\mathcal{C}(L, U) = \sum_{i=1}^N \sum_{j=1}^N l_{ij} \ln\left(\frac{l_{ij}}{u_{ij}}\right) \quad (5)$$

among all the matrices fulfilling (4) with the convention that  $l_{ij} = 0$  whenever  $u_{ij} = 0$  and  $0 \ln(\frac{0}{0})$  is defined to be 0.

The constraints for the estimations of the matrix  $L$  are not always consistent. For instance the liabilities of all banks in sector  $k$  against all banks in sector  $l$  do typically not equal the claims of all banks in sector  $l$  against all banks in sector  $k$ . We deal with this problem by applying a two step procedure.

In a first step we replace an a priori matrix  $U$  reflecting only possible links between banks by an a priori matrix  $V$  that takes actual exposure levels into account. As there are seven sectors we partition  $V$  and  $U$  into 49 sub-matrices  $V^{kl}$  and  $U^{kl}$  which describe the liabilities of the banks in sector  $k$  against the banks in sector  $l$  and our a priori knowledge. Given the bank balance sheet data we define  $u_{ij} = 1$  if bank  $i$  belonging to sector  $k$  might have liabilities against bank  $j$  belonging to sector  $l$  and  $u_{ij} = 0$  otherwise. The (equality) constraints are that the liabilities of bank  $i$  against the sector  $l$  equal the row sum of the sub-matrix and that the claims of bank  $j$  against the sector  $k$  equal the column sum of the sub-matrix, i.e.

$$\sum_{j \in l} v_{ij} = \text{liabilities of bank } i \text{ against sector } l \quad (6)$$

$$\sum_{i \in k} v_{ij} = \text{claims of bank } j \text{ against sector } k \quad (7)$$

For the matrices describing claims and liabilities within a sector (i.e.  $V^{kk}$ ) which has a central institution we get further constraints. Suppose that bank  $j^*$  is the central institution. Then

$$v_{ij^*} = \text{liabilities of bank } i \text{ against central institution} \quad (8)$$

$$v_{j^*i} = \text{claims of bank } i \text{ against central institution} \quad (9)$$

Though these constraints are inconsistent given our data, we use the information to get a revised matrix  $V$  which reflects our a priori knowledge better than the initial matrix  $U$ . Contrary to  $U$  which consists only of zeroes and ones, the entries in  $V$  are adjusted to the actual exposure levels.<sup>16</sup>

In a second step we recombine the results of the 49 approximations  $V^{kl}$  to get an entire  $N \times N$  improved a priori matrix  $V$  of inter-bank claims and liabilities. Now we replace the original constraints by just requiring that the sum of *all* (inter-bank) liabilities of each bank equals the row sum of  $L$  and the sum of *all* claims of each bank equals the column sum of  $L$ .

$$\sum_{j=1}^N l_{ij} = \text{liabilities of bank } i \text{ against all other banks} \quad (10)$$

$$\sum_{i=1}^N l_{ij} = \text{claims of bank } j \text{ against all other banks} \quad (11)$$

Again we face the problem that the sum of all liabilities does not equal the sum of all claims but corresponds to only 96% of them. By scaling the claims of each bank by 0.96 we enforce consistency.<sup>17</sup> Given these constraints and the prior matrix  $V$  we estimate the matrix  $L$ .

Finally we can use the information on claims and liabilities with the central bank and with banks abroad. By adding two further nodes and by appending the rows and columns for these nodes to the  $L$  matrix, we get a closed (consistent) system of the inter-bank network.

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<sup>16</sup>Note that the algorithm that calculates the minimum entropy entries does not converge to a solution if data are inconsistent. Thus to arrive at the approximation  $V$  we terminate after 10 iterations immediately after all row constraints are fulfilled.

<sup>17</sup>The remaining 4% of the claims are added to the vector  $e$ . Hence they are assumed to be fulfilled exactly.